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Linear and Non-linear Analysis of Beams on Elastic Foundation with and without Stone Columns by Parameterized Algorithm

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ABSTRACT: *This paper introduced a Modern Computational and Numerical Method in the form of Parameterized Gaussian Elimination Algorithm developing by a programme in C++ language which deals with linear and non-linear analysis of a typical one bay footing acting as a finite beam resting on granular bed – reinforced soft soil and stone columns . Granular fill has been idealized as Pasternak shear layer. Soft soil has been idealized by nonlinear Kelvin-Voigt body and the stone columns have been modeled as series of nonlinear Winkler springs. Non-linear behavior of granular layer and soft soil has been incorporated by hyperbolic constitutive relationships. It has been observed that deflection of footing increases in the case of linear analysis and decreases in case of non-linear analysis. Settlement of the footing increases without stone columns.*
Keywords – *Gaussian Elimination Algorithm, Linear and Non linear behavior, Pasternak shear layer, Soft soil, Stone column.*

1. INTRODUCTION

There are from several years' modern computational and numerical methods available in the field of Civil Engineering i.e. Finite Difference Method, Finite Element Method, LU Decomposition, Gauss Seidal, Gaussian Elimination which are applicable for modeling and analysis of finite and infinite beams with Linear and Non-linear analysis. Gaussian Elimination Technique is one of the most important, most precisely and widely used algorithm which can be developed by C/C++ Programming , FORTRAN Language or in MAT LAB. Various research workers have modeled and analyzed beams on elastic foundation treated as ground – foundation system [1-4]. Some of the studies did not consider the granular fill layer which is usually provided at the top of treated ground. Studies which considered the granular fill layer, did not consider the nonlinear behavior of soil [5-6]. Further, no study accounted for finite flexural rigidity of the foundation beam and provision of stone columns [7-8]. Provision of stone columns enhances the bearing capacity of the foundation and reduces the settlement.

In view of the available literature, it was felt that there is a need for analysis of footings having finite flexural rigidity resting on granular bed soft soil system. Maheshwari and Khatri [9] presented an analysis to address such a problem which had very limited parametric analysis with footing supporting two columns resting on poor foundation soil system. Therefore, in the present paper a parametric study has been carried out by developing a Parameterized Gaussian Elimination Algorithm to analyze Linear and Non-linear behavior of combined footing supporting two columns with soft soil foundation system and with series of stone columns.

2. PROPOSED SOIL FOUNDATION SYSTEM

Figure 1 shows combined footing idealized as an elastic beam (flexural rigidity *EI*) of length, 2*L*, acted upon by two concentrated loads (Q_1, Q_2,) and resting on the surface of a granular fill layer of thickness, *H* and shear modulus, *G*, placed over a stone column treated natural soft soil deposit. The diameter and spacing of stone columns is *d* and *s* respectively. Figure 2 shows the granular fill layer has been idealized as nonlinear Pasternak shear layer. The foundation soil has been idealized by nonlinear Kelvin-Voigt body and the stone columns have been modeled as series of nonlinear Winkler springs. The objective of the study is to first develop the governing differential equations guiding the flexural response of the foundation resting on stone column treated ground so that it can be designed accordingly.

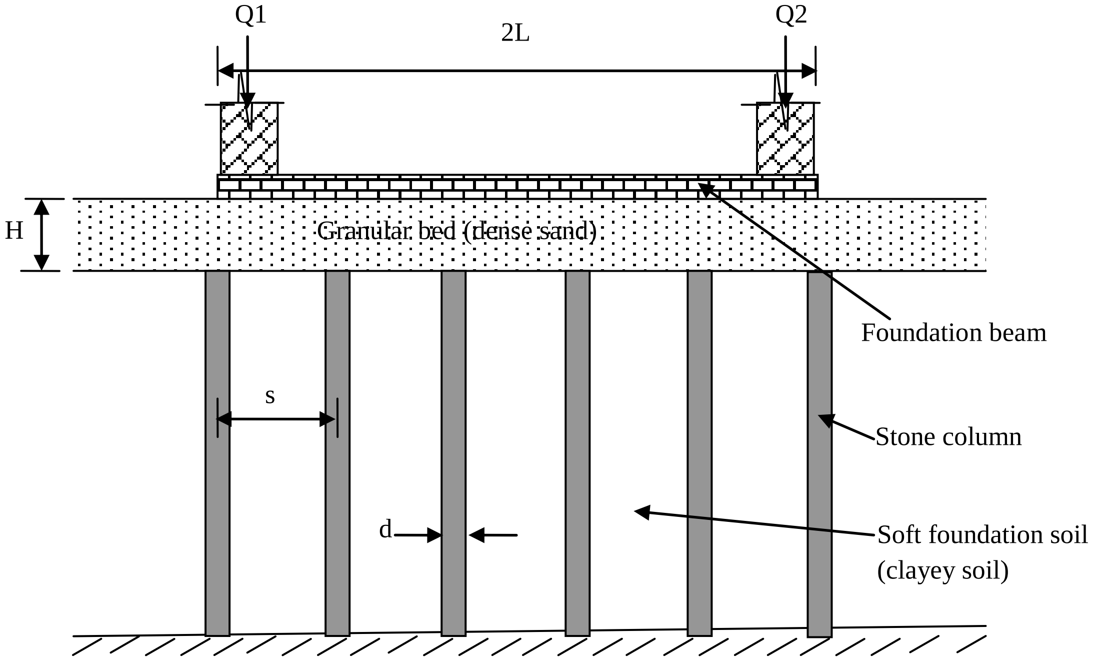

Fig. 1: Definition Sketch of the Problem

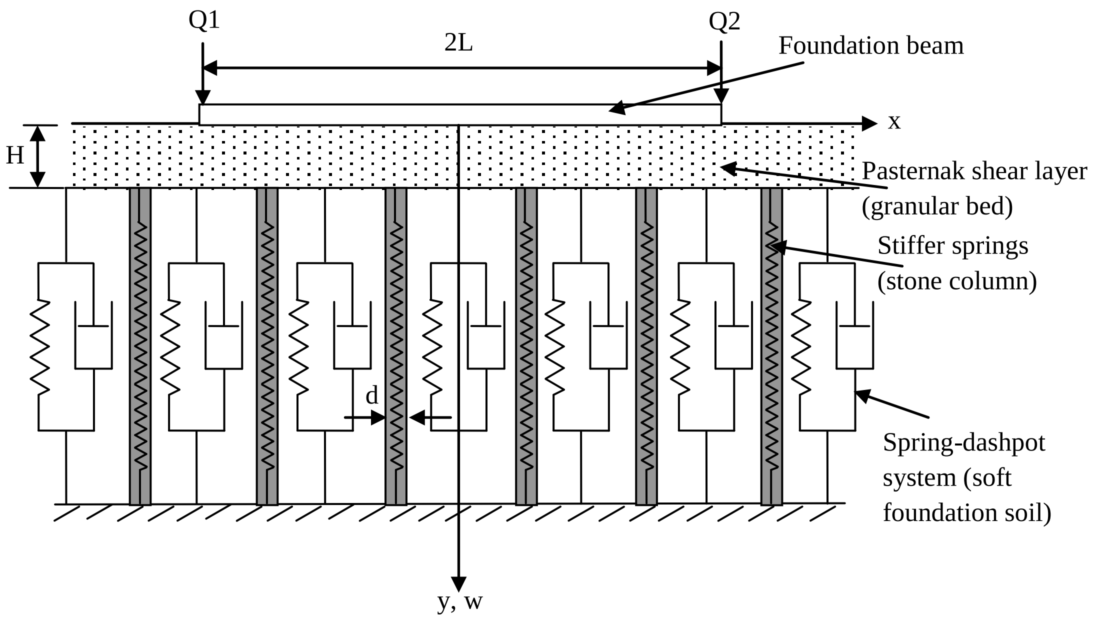

Fig. 2: Idealization of the Problem

3. ANALYSIS

A hyperbolic nonlinear stress-displacement relationship proposed by Kondner and Zelasko [10] has been considered to exhibit the behavior of granular fill and stone column. The stress – displacement response of the saturated soft soil has been represented by a hyperbolic relation as proposed by Kondner (1963). Stone columns have been assumed to be installed throughout the depth of natural soil bed overlying a rigid stratum.

The soft soil has been assumed to be in saturated condition. According to free body diagram for the granular fill layer (idealized as Pasternak shear layer) , The vertical force equilibrium equation for this granular fill layer can be written as –

$$q = q_s - G\,H\,\frac{d^2 w}{dx^2} \tag{1}$$

where, q is the reaction of granular fill on beam; q_s, the vertical force interaction between granular shear layer and the saturated soft foundation soil; w, the vertical deflection and x, be the coordinate along the length of the foundation beam.The shear modulus of granular layer can be expressed by considering the hyperbolic shear stress-shear strain response (Ghosh and Madhav, 1994) as –

$$G = \frac{G_o}{\left[1 + \frac{G_o\left|dw/dx\right|}{\tau_u}\right]^2} \tag{2}$$

where, G_o is the initial shear modulus of shear layer and τ_u is the ultimate shear resistance of the granular layer. The vertical force interaction between granular shear layer and the saturated soft foundation soil, q_s at any time t > 0, can be expressed employing effective stress principle as –

$$q_s = \overline{\sigma} + u_e \tag{3}$$

where, $\overline{\sigma}$ and u_e is average effective stress and average excess pore water pressure at time, t in the spring dashpot system respectively. Considering the hyperbolic nonlinear stress-displacement relationship (Kondner, 1963), $\overline{\sigma}$ can be expressed as –

$$\overline{\sigma} = \frac{k_{so}\,w}{1 + k_{so}\,(w/q_u)} \tag{4}$$

where, k_{so} is the initial modulus of subgrade reaction and q_u, the ultimate bearing resistance of saturated soft soil respectively.

Combining equations (3) and (4), one gets –

$$q_s = \frac{k_{so}\,w}{1 + k_{so}\,(w/q_u)} + u_e \tag{5}$$

The average excess pore water pressure at any time, t can be expressed as:

$$u_e = u_o\,(1 - U) \tag{6}$$

where, u_o is the initial pore water pressure and U, the average degree of consolidation at time, t which is due to vertical (U_v) as well as radial drainage (U_r) . Combining equations (5) and (6) and initially, i.e., at time t = 0, stress at the interface of granular fill and saturated soft soil is carried by the excess pore water pressure within the surrounding soil. In view of this, equation can be rewritten as –

$$q_s = \frac{k_{so}\,w}{U\left[1 + k_{so}\,(w/q_u)\right]} \tag{7}$$

The vertical force interaction between granular shear layer and the saturated soft foundation soil, q_c can be written as (Kondner and Zelasko, 1963) –

$$q_c = \frac{k_{co}\, w}{1 + k_{co}\,(w / q_{cu})} \tag{8}$$

where, k_{co} and q_{cu} are initial modulus of subgrade reaction and the ultimate bearing resistance of stone columns respectively. The reaction of granular fill on beam can therefore be written as –

$$q = \frac{k_{so}\, w}{U\left[1 + k_{so}\,(w / q_u)\right]} - G\,H\,\frac{d^2 w}{dx^2} \text{, within saturated soft soil region} \tag{9}$$

and,

$$q = \frac{k_{co}\, w}{1 + k_{co}\,(w / q_{cu})} - G\,H\,\frac{d^2 w}{dx^2} \text{, within the stone column region} \tag{10}$$

The differential equation of a beam can be obtained by considering the bending of an elemental segment.

$$EI\,\frac{d^4 w}{dx^4} + \frac{k_{so}\, w}{U\left[1 + k_{so}\,(w / q_u)\right]} - G\,H\,\frac{d^2 w}{dx^2} = 0 \text{, within saturated soft soil region} \tag{11}$$

and,

$$EI\,\frac{d^4 w}{dx^4} + \frac{k_{co}\, w}{\left[1 + k_{co}\,(w / q_{cu})\right]} - G\,H\,\frac{d^2 w}{dx^2} = 0 \text{, within stone column region} \tag{12}$$

where, EI is the flexural rigidity of the beam and p, the externally applied load intensity=0.

4. GOVERNING DIFFERENTIAL EQUATIONS

The governing differential equations have been expressed in the non-dimensional form employing the following non-dimensional parameters:

$X = x / L$, $W = w / L$, $G^* = G\,H / k_{so} L^2$, $G_o^* = G_o\,H / k_{so} L^2$, $I^* = EI / k_{so} L^4$, $q_u^* = q_u / k_{so} L$, $q_{cu}^* = q_{cu} / k_{co} L$, $\tau_u^* = \tau_u\,H / k_{so} L^2$, $Q^* = Q / k_{so} L^2$ and $\alpha = k_{co} / k_{so}$.

$$\frac{d^4 W}{dX^4} + \frac{1}{I^*}\,\frac{W}{U\left[1 + \left(W / q_u^*\right)\right]} - \frac{G^*}{I^*}\,\frac{d^2 W}{dX^2} = 0 \tag{13}$$

$$\frac{d^4 W}{dX^4} + \frac{\alpha}{I^*}\,\frac{W}{1 + \left(W / q_{cu}^*\right)} - \frac{G^*}{I^*}\,\frac{d^2 W}{dX^2} = 0 \tag{14}$$

A finite difference scheme has been employed to solve the governing differential equations (13) and (14) of the soil-foundation system under consideration. The equations can be written in a finite difference form for an interior node, i, at any time, t > 0 with in soft soil foundation region and stone column region respectively as:

$$W_{i-2} + W_{i-1}\left[-4 - \frac{G*}{I*}(\Delta X)^2\right] + W_i\left[6 + \frac{2G*}{I*}(\Delta X)^2 + \frac{1}{I*}\frac{(\Delta X)^4}{U\{1+(W_i/q_u*)\}}\right] + W_{i+1}\left[-4 - \frac{G*}{I*}(\Delta X)^2\right] + W_{i+2} = 0 \tag{15}$$

$$W_{i-2} + W_{i-1}\left[-4 - \frac{G*}{I*}(\Delta X)^2\right] + W_i\left[6 + \frac{2G*}{I*}(\Delta X)^2 + \frac{\alpha}{I*}\frac{(\Delta X)^4}{\{1+(W_i/q_{cu}*)\}}\right] + W_{i+1}\left[-4 - \frac{G*}{I*}(\Delta X)^2\right] + W_{i+2} = 0 \tag{16}$$

Due to symmetry, only half of the soil-foundation system has been considered in the analysis. The boundary conditions can be written in non-dimensional form as –

$$\text{At X = 0, } \frac{dW}{dX} = 0 \quad \text{and} \quad \frac{d^3W}{dX^3} = 0, \tag{17}$$

$$\text{At X = 1, } \frac{d^2W}{dX^2} = 0 \quad \text{and} \quad \frac{d^3W}{dX^3} - \frac{G*}{I*}\frac{dW}{dX} = \frac{Q*}{I*} \tag{18}$$

The governing differential equations (15) and (16) have been solved along with appropriate boundary conditions (17) and (18) using Gauss Elimination iterative scheme. The solutions have been obtained with convergence criteria as -

$$\left|\frac{W_i^k - W_i^{k-1}}{W_i^k}\right| \times 100\% < \varepsilon_s$$

For all *i*, where *k* and *k*-1 are the present and previous iterations respectively and ε_s is the specified tolerance which has been considered to be 10^{-5} in the present study.

5. RESULTS AND DISCUSSIONS

Based on the above formulation, a computer program was developed using finite difference scheme. Due to symmetry, half length of the beam (*L*) was discretized finite difference wise and it was observed that the difference in response corresponding to finite difference mesh with 101 nodes and 201 nodes was less than 1.0% and hence the mesh with 101 nodes was considered for all parametric studies. The range for values of various parameters has been presented in Table 1 in dimensional form and non-dimensional range has been presented in Table 2. In this paper linear and non-linear analysis, with and without stone columns has been emphasized.

TABLE 1 - Range of values of various parameters considered for parametric study

Parameter	***Symbol***	***Range of values***	***Unit***
Applied load	*Q*	50 – 200	kN
Flexural Rigidity of footing	*EI*	15 – 300	MN-m^2

Half length of footing	L	2.5	m
Thickness of granular fill layer	H	0.3	m
Diameter of stone columns	d	0.2 – 0.4	m
Spacing to diameter ratio for stone columns	s / d	2.5 – 4	-
Initial modulus of subgrade reaction for soft soil	k_{so}	10 **[11-12]**	MN/m^2
Initial shear modulus of granular fill	G_o	652.4 **[13]**	kN/m^2
Ultimate bearing resistance of soft foundation soil	q_u	20 – 60	kN/m^2
Ultimate bearing resistance of stone column	q_{cu}	100 – 200	kN/m^2
Ultimate shear resistance of granular fill layer	τ_u	4 – 10	kN/m^2
Relative stiffness of stone column	α	10 – 100	-
Average degree of consolidation	U	40 – 100%	-

Table 2 - Range of values of non-dimensional parameters considered for parametric study

Non-dimensional parameter	***Expression***	***Range of values***
Q^*	$Q / k_{so} L^2$	$8 \times 10^{-4} – 3.2 \times 10^{-3}$
I^*	$E I / k_{so} L^4$	0.0384 – 0.768
G_o^*	$G_o H / k_{so} L^2$	0.0313
q_u^*	$q_u / k_{so} L$	$8 \times 10^{-4} – 2.4 \times 10^{-3}$
q_{cu}^*	$q_{cu} / \alpha k_{so} L$ (α=25)	$1.6 \times 10^{-4} – 3.2 \times 10^{-4}$
τ_u^*	$\tau_u H / k_{so} L^2$	$1.92 \times 10^{-5} – 4.8 \times 10^{-5}$

Comparison of results for linear and nonlinear response of soil – foundation system

A comparison between linear and the nonlinear analysis of foundation beam resting on granular fill – stone column improved soil system has been depicted in Fig. 3. Typically, the input parameters have been considered as I^* = 0.3968, G_o^* = 0.0313, Q^* = 3.2×10^{-3}, d/L = 0.12, s/d = 3, q_u^* = 1.6×10^{-3}, q_{cu}^* = 2.4×10^{-4}, τ_u^* = 3.36×10^{-5}, α = 25 and U = 100%. The figure shows a reduction of 95.65% in the maximum deflection of foundation beam from nonlinear response to linear response.

Comparison of results for response of soil – foundation system with and without stone columns

Deflection of the foundation beam has been found to reduce by 97.52% in case the saturated soft soil is reinforced with stone columns for the values of input parameters as mentioned in Fig. 4.

Influence of applied load (Q)

The effect of applied load (at the edge of footing) on deflection has been shown in Fig. 5 for typical values of input parameters as $I^* = 0.384$, $G_o^* = 0.0313$, $d/L = 0.12$, $s/d = 3$, $q_u^* = 1.6\times10^{-3}$, $q_{cu}^* = 2.4\times10^{-4}$, $\tau_u^* = 3.36\times10^{-5}$, $\alpha = 25$ and $U = 90\%$. It has also been attempted to find out the maximum load carrying capacity of the foundation beam for typical values of input parameters and it has been observed that maximum load carrying capacity increases from about 1.52×10^{-3} to 3.36×10^{-3} with the inclusion of stone columns.

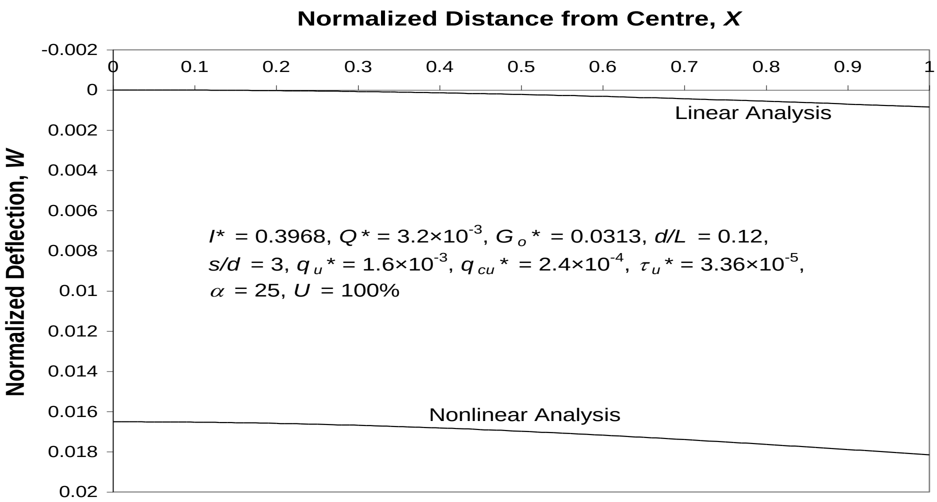

Fig. 3 Comparison of deflection profiles of foundation beam for linear and nonlinear analysis

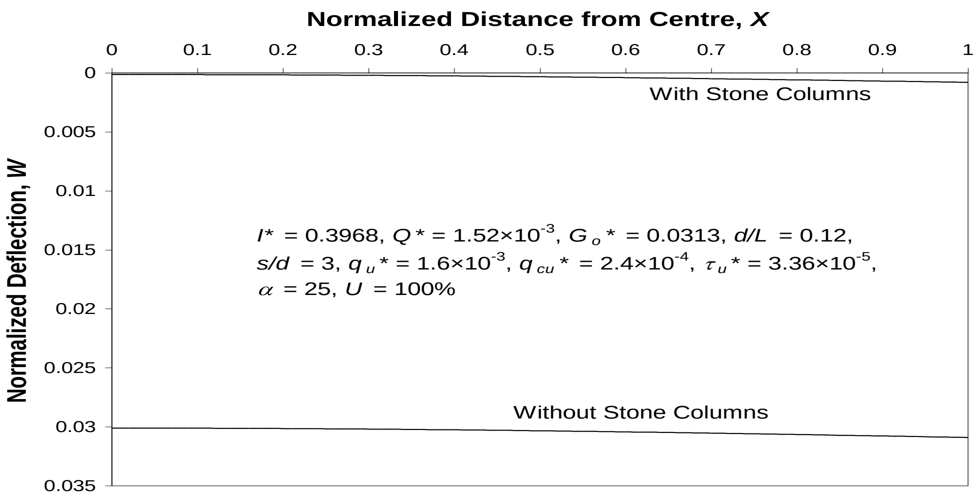

Fig. 4 Deflection profiles of foundation beam with and without stone columns

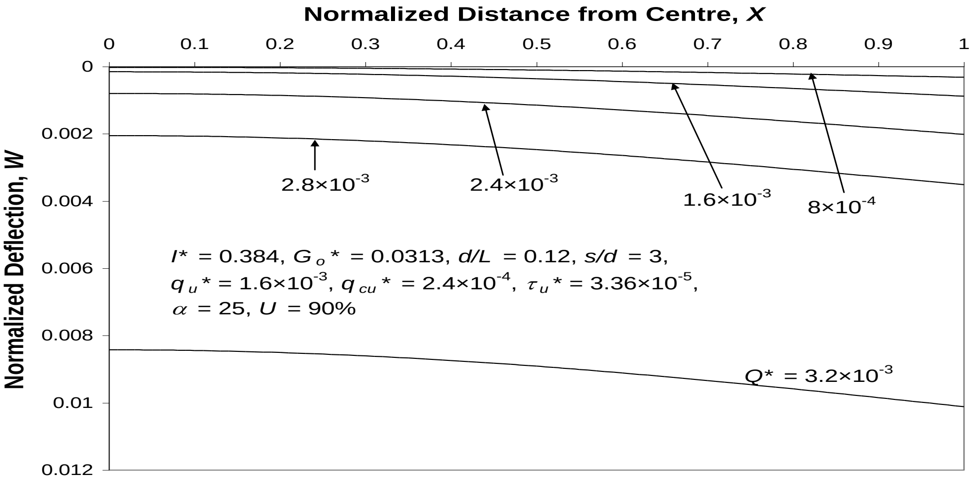

Fig. 5 Influence of applied load on normalized deflection of foundation beam

6. CONCLUSIONS

The proposed model has been successfully employed for the study of flexural behavior of footings resting on granular fill – stone column improved saturated soft foundation soil. Nonlinear behavior of granular fill, stone columns and the soft soil has been considered in the analysis by means of hyperbolic constitutive relationships.

Based on a detailed parametric study as presented above, the following conclusions have been drawn:

i) As expected, the deflection of foundation beam reduces with employment of stone columns in the soft foundation soil. The maximum deflection at the edge of foundation beam has been found to reduce by about 98%. Further, the maximum load carrying resistance also enhances from 1.52×10^{-3} to 3.36×10^{-3} with the inclusion of stone columns for typical values of input parameters.

ii) The response of the soil – foundation system is greatly affected by considering nonlinear behavior of soils. For particular values of input parameters, the maximum deflection of the footing has been found to reduce by 96% for linear analysis as compared to the nonlinear analysis.

iii) Maximum normalized deflection has been found to reduce by 97 % as the normalized load reduces from 3.2×10^{-3} to 8×10^{-4} and the corresponding reduction in maximum normalized bending moment (at the centre of beam) has been found to be around 83 %. It has been observed that the beam can be subjected to a maximum load of about 3.36×10^{-3} for typical values of other input parameters considered. As applied load approaches to the failure load, rate of increase of deflection of beam has been observed to be higher.

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